Lead & Lag Compensators

* when a plant needs a controller, the controller will move the poles of the system to the desired location.

* root locus method shows where the zeros and the poles of the system.

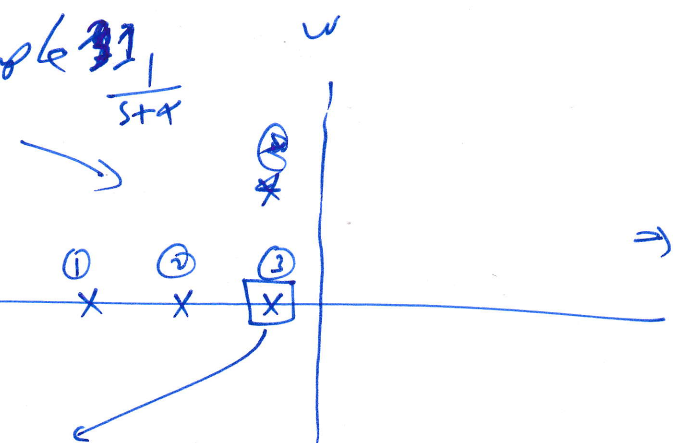
* the compensator comes in shape of transfer function (G_{com}): $G_{com} = \frac{(s-a_1)(s-a_2)\cdots(s-a_n)}{(s-b_1)(s-b_2)\cdots(s-b_m)}$

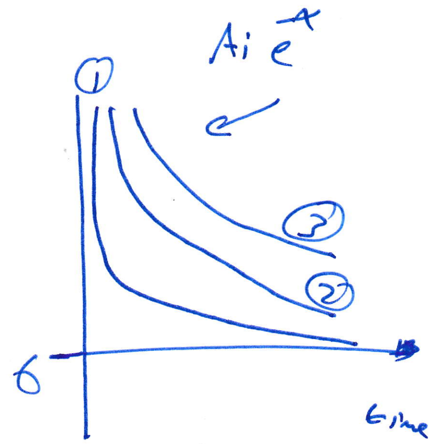
where the numbers of n & m are determined based on the control requirements

* in Root locus method as controlling method, the dominant poles are the one important.

Example 1, $\frac{1}{s+\alpha}$

For 1st order syst

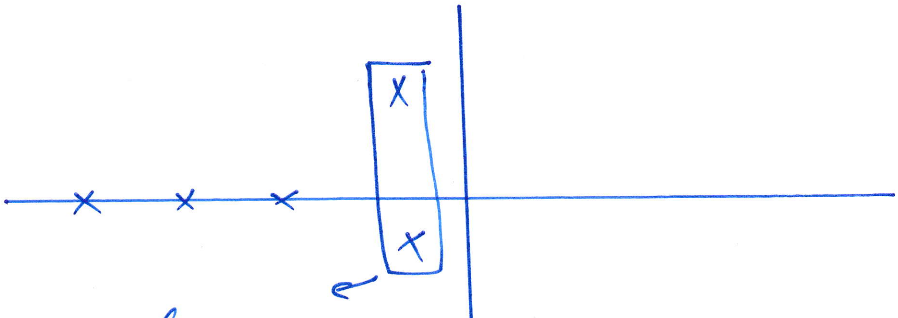




the poles near the imaginary axis are the dominate.

3 is the slowest response, then it is the dominant pole.

For systems with ~~a~~ more than 2^{nd} order (i.e. third order or higher), the dominat Poles may Reduce the problem to 2^{nd} order System if the number dominate ~~ro~~ Poles ~~are~~ are 2

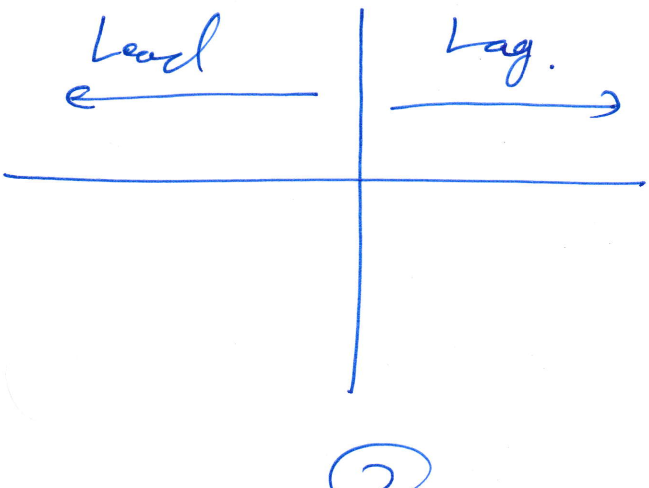


Dominate ~~ro~~ Poles
make this system as 2^{nd} order system

For 2^{nd} order system, the concept of damping is ~~more~~ easier

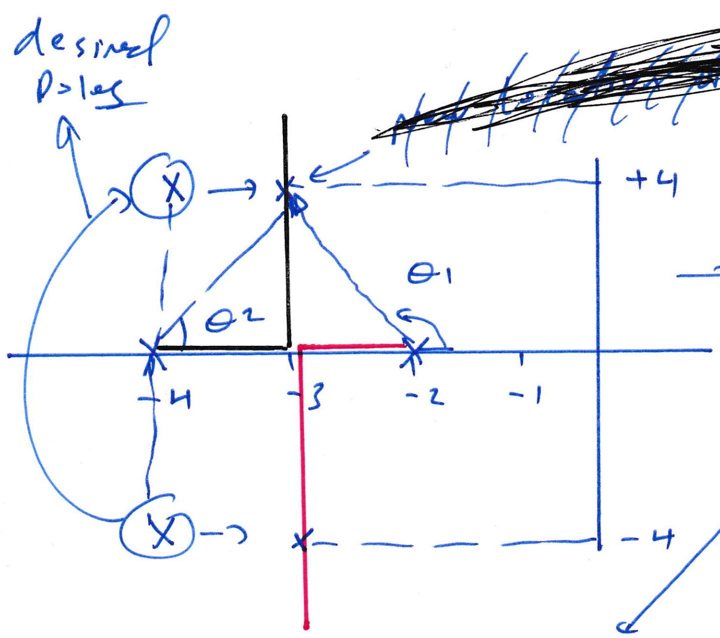
$$t_r \cdot \omega_n = \frac{1}{\sqrt{1-\zeta^2}}\left(\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

A Lead Compensator moves the dominate Poles ~~poles~~ ~~roots~~ to the left while the Lag Compensator moves the poles to the righ side of s-plane.



Example. 2

$$G(s) = \frac{1}{(s+2)(s+4)}$$



$\rightarrow \theta_1 + \theta_2 + 0 = 180$

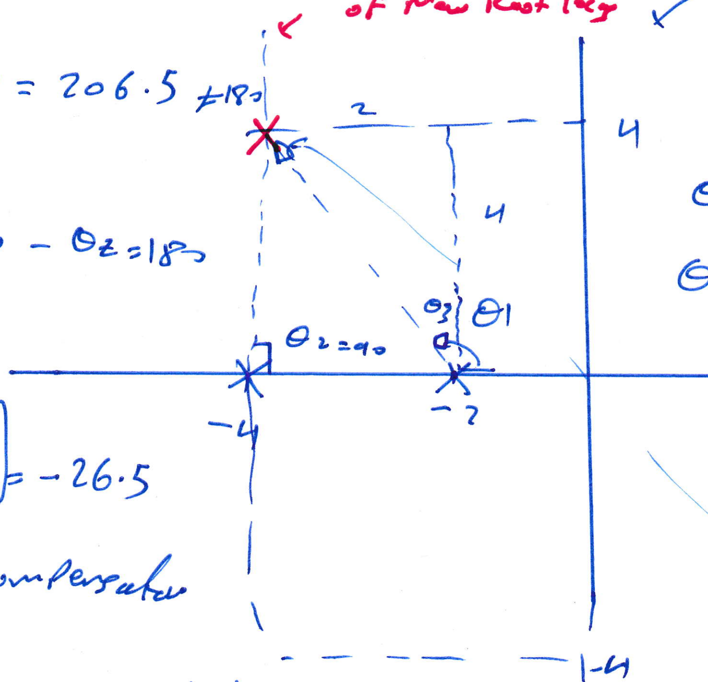
Angle Condition $\sum \angle p - \sum \angle z = 180$

$\hookrightarrow \theta_2 + \theta_1 = 206.5 \neq 180$

$206 + \theta_p - \theta_z = 180$

the position of New Root locus

the desired situation



$\theta_1 = 90 + \theta_3$

$\theta_3 = \tan^{-1} \frac{2}{4} = 26.5$

$\theta_1 = 116.5$

$\Rightarrow \boxed{\theta_p - \theta_z} = -26.5$

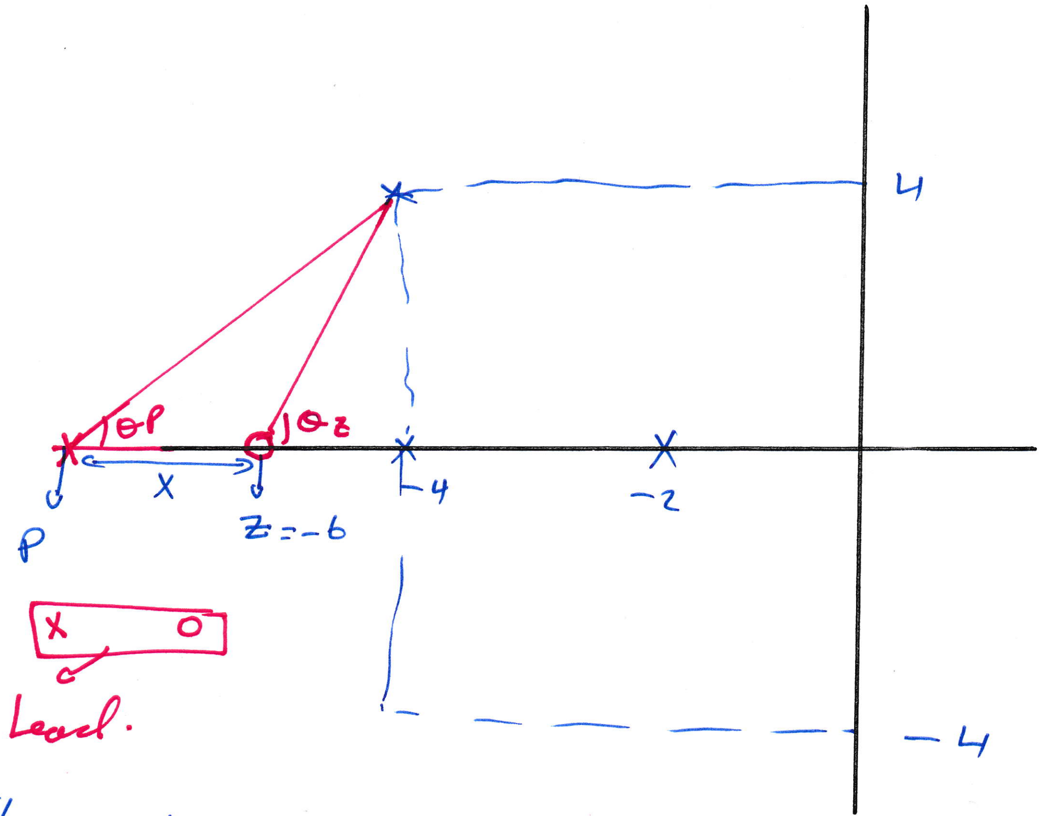
$\hookrightarrow$ lead Compensator

$\hookrightarrow$ Chose an arbitary Zero for the Compensator

Ass $\theta_p - \theta_z = -ve$ then $\theta_z > \theta_p$

which mean the Zero is near to the Pole (X) than the Pole.

$\theta_p - \theta_z = -26.5$

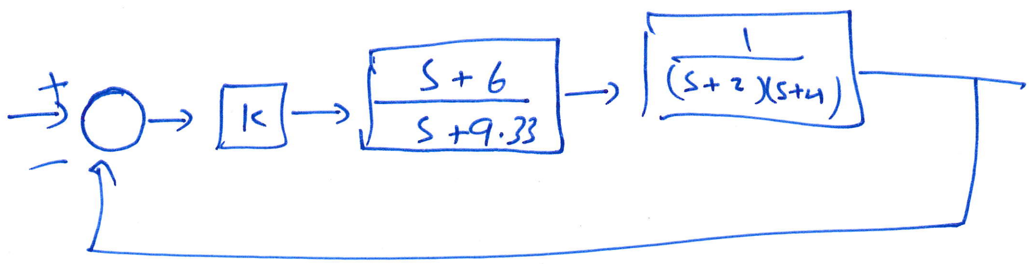


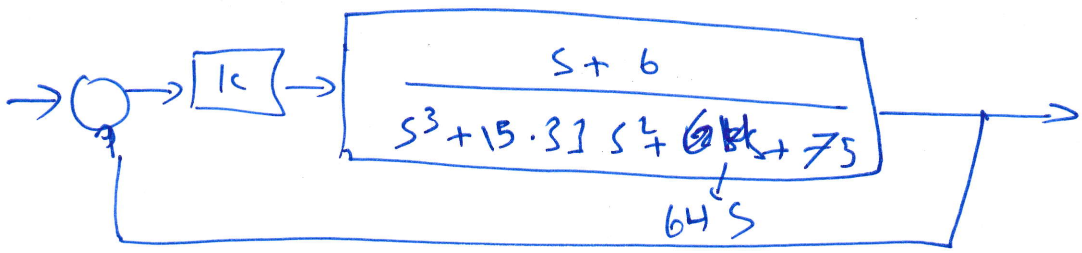
Chose the zero $Z = -6 \Rightarrow \theta_z = \tan^{-1}\frac{4}{2} = 63.43$

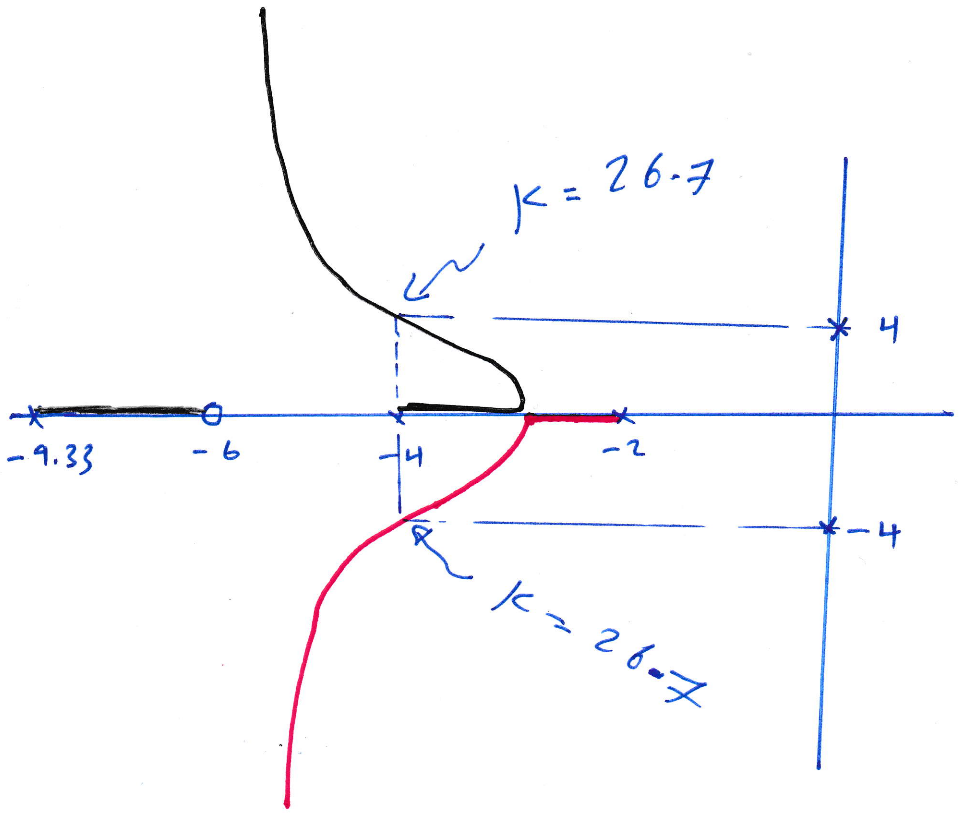
$\Rightarrow \theta_p = 63.43 - 26.5 = 36.93 = \tan^{-1}\frac{4}{2+x}$

$\Rightarrow \frac{4}{2+x} = 0.75 \Rightarrow 1.5 + 0.75x = 4 \Rightarrow x = 3.33$

$P = -9.33$







Lag Compensator:-

Back to e_{ss}:- For step input

$$e_{ss} = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+\lim_{s\to 0}\frac{N(s)}{D(s)}} = \frac{1}{1+\frac{N(0)}{D(0)}}$$

$$\Rightarrow e_{ss} = \frac{\frac{D(0)}{\cancel{D(0)}}}{\frac{D(0)+N(0)}{\cancel{D(0)}}} = \frac{D(0)}{D(0)+N(0)}$$

Add Compensator $\frac{S-Z}{S-P} \Rightarrow G(s) = \frac{(s-z)\,N(s)}{(s-p)\,D(s)}$

$$\Rightarrow e_{ss} = \frac{D(0)\cdot P}{D(0)\cdot P + N(0)\,Z} \Rightarrow \frac{Z}{P} = \frac{D(0) - e_{ss}\,D(0)}{e_{ss}\,N(0)}$$

to have $e_{ss} = 0 \Rightarrow \frac{Z}{P} = \infty$ which seem impossible.

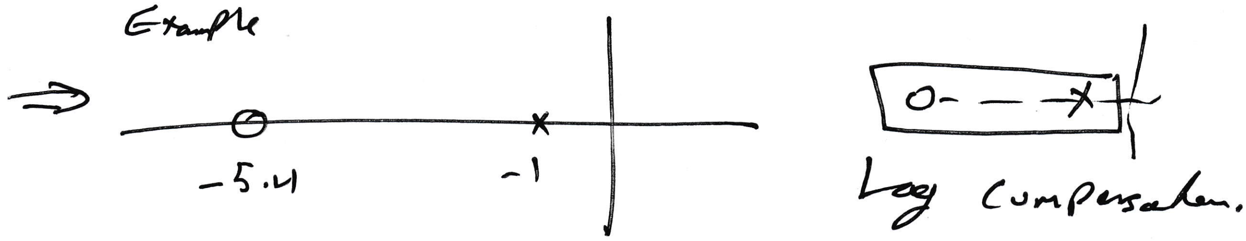
Example 3 on Using Compensator to satisfy the steady state error condition.

Assume $G(s) = \dfrac{s+10}{s^2+5s+6}$ and $e_{ss} = 0.1$

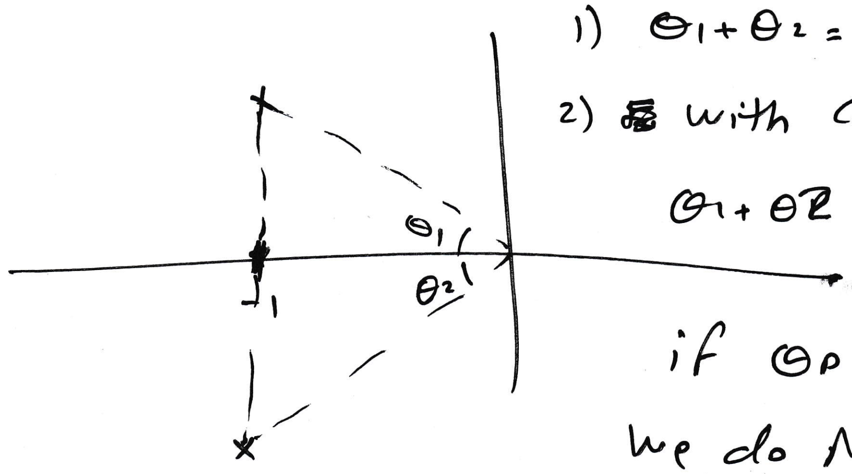
$$\frac{Z}{P} = \frac{D(0) - e_{ss} D(0)}{e_{ss} N(0)} = \frac{6 - 6(0.1)}{(0.1)(10)} = 5.4$$

this means $Z = 5.4\,P$ and if the pole (P) is Negative, then the zero is going far in the Negative Real axis by 5.4.

Example



if the system Needed to be controlled is



1) $\theta_1 + \theta_2 = 180$

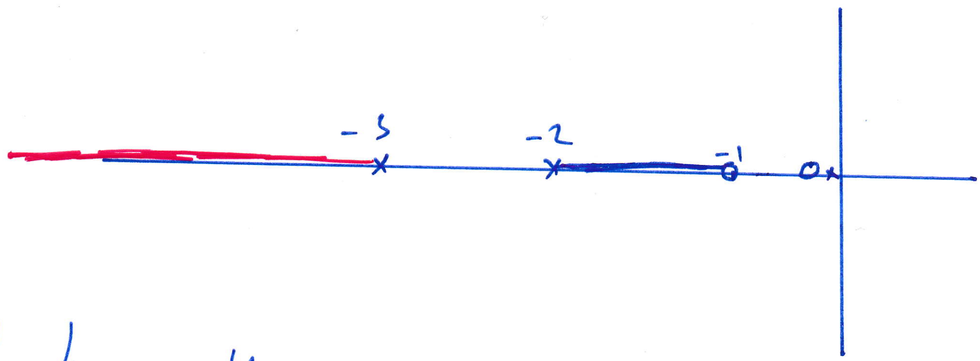
2) with Compensator

$$\theta_1 + \theta_2 + \theta_p - \theta_z = 180$$

if $\theta_p - \theta_z \approx 0$

we do Not have to change the Root locus. this can be done if (Z) location $= \dfrac{\text{Real Dominant Pole}}{50}$

For the system : $\frac{s+1}{s^2+5s+6}$



to have the $e_{ss} = 0.1$, then the $\frac{z}{p} = 5.4$

Let the location of $z = \frac{-2}{50} = -0.04$

$p = -7.4 \times 10^{-3}$

Back to the Example ②, when the Lead Compensator is added, $G = \frac{s+6}{s^3+15.33s^2+64s+75}$

For step input.

$$e_{ss} = \frac{D(0)}{D(0)+N(0)} = \frac{75}{75+6} = 0.93$$

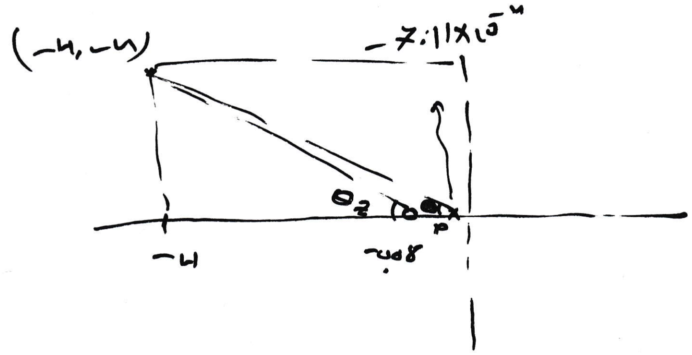
which too Large. To Reduce this error the Lag compensator is used with the Previous Lead Compensator

$$\frac{Z}{P} = \frac{D(0) - e_{ss} D(0)}{e_{ss} N(0)}$$ assume $e_{ss} = 0.1$

$$\Rightarrow \frac{Z}{P} = \frac{75 - (0.1)(75)}{(0.1)(6)} = 112.5$$

$$Z = \frac{-4}{50} = -0.08 \rightarrow P = -7.11 \times 10^{-4}$$

As seen, the additional zero and pole are too small and produce small angle diffrence ($\theta_P - \theta_Z$)



$$\theta_P = \tan^{-1} \frac{4}{4 - 7.11 \times 10^{-4}} \simeq 45 \qquad \theta_Z = \tan^{-1} \frac{4}{4 - 0.08} = 45.5$$

$\Rightarrow \theta_P - \theta_Z \simeq -0.5$ which can be Neglicted which means: it does not effect the lead comp